Analytical orbital theory analysis of electron confinement in a Polywell™ device

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Aims for our analysis was to analytically derive expressions for:

1. Electron confinement scaling laws.

2. The average radial electron turning distances.

3. Power loss expressions for maintaining equilibrium electron density.

These expressions were derived from an orbit theory analysis or first principles approach. Comparisons will be made with previous orbit theory simulations.
Polywell™ a virtual cathode system

- Central minimum magnetic field has certain plasma M.H.D. Stability properties.
- Virtual cathode may form due to space charge trapping of electrons.
- Ions are then electrostatically confined by the electron’s electric field.

Fig. 1 Magnetic field structure inside Polywell™, highlighting the magnetic null.

Fig. 2 A three dimensional schematic layout of Buzzard's Polywell™ design.

Fig 1. Image: N. Krall, Physics of Plasmas, 2, 1995. Reformatted in colour by Mark Duncan, 2007
Magnetic null

**Fig. 1** Magnetic field structure inside Polywell™, highlighting the magnetic null.

- Spindle cusp generates a circular line cusp.
- Polywell effectively ‘plugs’ this line cusp.
- 2 point and 1 line cusp **VS** 14 point cusps.

**Fig. 3** Spindle cusp magnetic field line projection along with guiding center particle orbits.

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**Fig 3.** J. Egedal and A. Fasoli, Physics of Plasmas, 8, 2001.
Trapped electron motion
Five main parameters of interest:

<table>
<thead>
<tr>
<th>Param.</th>
<th>Description</th>
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<tbody>
<tr>
<td>R</td>
<td>Radius (m)</td>
</tr>
<tr>
<td>I</td>
<td>Current (A)</td>
</tr>
<tr>
<td>$\frac{dI}{dt}$</td>
<td>Rate of $\Delta I$ (A/s)</td>
</tr>
<tr>
<td>$K$</td>
<td>Energy (eV)</td>
</tr>
<tr>
<td>$s$</td>
<td>Coil spacing (m)</td>
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</tbody>
</table>

- Low Beta regime.
- Non-interacting condition.
Magnetic field for $I = 10^5$ A and $R = 10$ cm
Adiabatically invariant only if:

\[ \frac{v_L}{L_m} \ll \omega_c \]

where

\[ L_m \equiv \left| \frac{B}{\nabla B} \right| \]

Two equations:

\[ \omega_c \sim 10 \frac{v_L}{L_m} \quad (1) \]

\[ B(r) = ar^3 \]

\[ B_0 = \frac{35\mu_0 I}{64\sqrt{2}R^4} r_0^3 \quad (2) \]

\[ r_0 = 4R \left( \frac{3 \sin (1) \sqrt{Km/e}}{7\mu_0 I} \right)^{\frac{1}{4}}, \quad B_0 = \frac{5 (7\mu_0 I)^{\frac{1}{4}} \left( 3 \sin (1) \sqrt{Km/e} \right)^{\frac{3}{4}}}{\sqrt{2}R} \]

Eqn. 2: M. Carr, D. Gummersall, S. Cornish and J. Khachan, Physics of Plasmas, 18, 2011
Assume magnetic cusp acts like a 1D magnetic mirror.
System was adiabatically invariant a short distance from the central magnetic null.

\[
\frac{dN}{dt} = -P_{\text{loss}} f N
\]

\[
P_{\text{loss}} = 1 - \sqrt{1 - \frac{B_0}{B_m}} \approx \frac{B_0}{2B_m}
\]

where \( B_m = B_f \times \frac{I}{R} \)

\( B_m \) is the maximum strength of the magnetic field and \( B_0 \) is the strength of the magnetic field in the weakest section of the adiabatically invariant path.

Using \( f = \frac{v}{2R_T} \) and \( R_T = \left( \frac{64\sqrt{2}B_0R^4}{35\sin^2(1)\mu_0I} \right)^{1/3} \)

\[
N(t) = \exp \left[ -3.71 \times 10^6 \times \sqrt{\mu_0} \left( \frac{q}{m_e} \right)^{1/4} \times \frac{K^{3/4}}{\sqrt{IR}} t \right]
\]
Confinement time analysis

\[ N_0(t) = \exp \left[ -2.69 \times 10^6 \times \frac{K^{\frac{3}{4}}}{\sqrt{IR}} t \right] \]

Fig. 4 A sample of fitted curves to a 1 m radius Polywell™ with 100 eV electrons where the line data were the best fit curves to corresponding data.

Fraction of electrons inside Polywell

Time constant/scaling law:

\[ t_n = 5 \times 10^{-7} \times \frac{\sqrt{IR^{\frac{3}{2}}}}{K^{\frac{3}{4}}} \]
Magnetic field for $R = I = 1$

$$B(r_a) = 10^{-7} \times \frac{I}{R} \left( -4.27 \times r_a^4 + 8.70 \times r_a^3 - 2.24 \times r_a^2 + 0.23 \times r_a \right)$$

where

$$r_a = \frac{r}{R}$$

Can invert $B(r_a)$ to get a close approximation of $r_a(B)$.

Use this to find a more accurate $R_T$ as $B_T$ is known via:

$$\frac{1}{2} \frac{m v_T^2}{B_0} = \frac{1}{2} \frac{m v^2}{B_T}$$

$$B_T = \frac{B_0}{\sin^2 (1)}$$
Comparisons with simulation data

From simulation data we get the time-weighted average electron radial distance. This can be interpreted as roughly $R_T$ as radial velocity is zero at turning point.

Used only the electrons which stayed within the Polywell for 10xT.O.F. in absence of magnetic fields.

For a given current and electron energy can determine the average radius of electron. Hence can predict if electrons will be confined.
Power loss defined by:

\[ P_l = -\frac{dN}{dt} K e \]

After substitutions:

Total power loss:

\[ P_l = N_{\text{max}} 3.2 \times 10^{-13} \frac{K^{\frac{7}{4}}}{\sqrt{I R^{\frac{3}{2}}}} \]

Power loss density:

\[ \rho_l = \rho_{\text{max}} 3.2 \times 10^{-13} \frac{K^{\frac{7}{4}}}{\sqrt{I R^{\frac{3}{2}}}} \]

Where \( N_{\text{max}} \) and \( \rho_{\text{max}} \) are the maximum electron total number and density, respectively.

For a 1 litre volume

\[ R = 4 \text{ cm} \]
\[ \rho_{\text{max}} = 10^{16} \text{ m}^{-3} \]
\[ K = 10 \text{ keV} \]
\[ I = 10^6 \text{ A.Turns} \]
\[ P_l \sim 200 \text{ kW} \]

• No space charge effects
• No scattering
• No application of Child-Langmuir Law
Current simulation is a 1D2V gyrokinetic, leapfrog, PIC modeling of point cusps.

Simulation improvements:

• Expand model to 2D3V dimensions.
• Include full 2D electron motion.
• Include electron-electron scattering.
• Explore electrostatic ‘plugging’ of cusps (Doland, 1994).
• Include ion interactions with electron plasma.